

科目：微積分 適用：電機系二

編號：331

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題
共 / 頁
第 / 頁

1. Prove: If f is the function defined by $f(x) = \begin{cases} 2x+1, & x \leq 0 \\ x^2-x, & x > 0 \end{cases}$ then

$\lim_{x \rightarrow 0} f(x)$ does not exist. (10 points)

2. (a) Find $\frac{dy}{dx}$ for $y = (x^3+1)(3x^2+2x-1)$. (10 points)

- (b) Find $\frac{d}{dt} \left(t^3 - \frac{t}{t^2-1} \right)$. (10 points)

- (c) Find $\frac{d}{dx} \left(\frac{1-\sec x}{\tan x} \right)$, hint: $\sec^2 x - \tan^2 x = 1$. (10 points)

3. The number $\sqrt{3}$ is a root of the equation $x^2 - 3 = 0$. Please estimate $\sqrt{3}$ by applying the Newton-Raphson method to the function

$f(x) = x^2 - 3$ starting at $x_1 = 2$. Hint: the Newton-Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \quad (10 \text{ points})$$

4. Find f to satisfy that $f''(x) = 6x - 2$, $f'(1) = -5$, and $f(1) = 3$. (10 points)

5. (a) Find $\frac{d}{dx} \left(\int_x^{2x} \frac{dt}{1+t^2} \right)$. (10 points)

- (b) Compute $\int x^2 e^{-x} dx$. (10 points)

6. Given two vectors $a = 2i + 3j + 2k$ and $b = i + 2j - k$. (10 points)

(a) Calculate the angle between a and b . (5 points)

(b) Find $\text{proj}_b a$. (5 points)

7. Find an equation $f(x, y, z) = 0$ for the plane that passes through the

point $P(-2, 3, 5)$ and is perpendicular to the line l with scalar parametric equations: $x = -2 + t$, $y = 1 + 2t$, $z = 4$. (10 points)