

科目：工程數學甲 適用：電機所電子組

考生注意：
 1. 依次序作答，只要標明題號，不必抄題。
 2. 答案必須寫在答案卷上，否則不予計分。
 3. 限用藍、黑色筆作答；試題須隨卷繳回。

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Each of the following problems weighs 15 points except for Problem 6), which weighs 10 points.

1) The Fourier Transform of the sequence $x(n)$ is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega}$$

provided that the sequence is absolutely summable. That is $\sum_{n=-\infty}^{\infty} |x(n)| < \infty$. Suppose for two absolutely summable sequences $x(n)$ and $y(n)$ their supports do not overlap, namely $I_x \cap I_y = \emptyset$ where $I_x := \{n \mid x(n) \neq 0\}$ and $I_y := \{n \mid y(n) \neq 0\}$. Find

$$\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

where Y^* is the complex conjugate of Y .

2) The Fourier Transform of a signal $x(t)$ is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt.$$

Find $\varphi(t)$ if its Fourier Transform $\Phi(f) = \max\{0, 1 - |f - a|\}$, a is some real number.

3) Solve the following integral equation

$$\phi(t) + \cos t \int_0^t \phi(\xi) \cos \xi d\xi + \sin t \int_0^t \phi(\xi) \sin \xi d\xi = \sin 2t.$$

4) Find the 2×1 vector $x(t)$ that satisfies:

$$\frac{dx(t)}{dt} = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} x(t) + \begin{bmatrix} t^{-3} \\ t^{-2} \end{bmatrix}, \quad x(2) = \begin{bmatrix} 47/8 - 2 \ln 2 \\ 23/2 - 4 \ln 2 \end{bmatrix}$$

5) Find a series solution ($x > 0$) for the following differential equation.

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} - y = 0.$$

6) The Laplace transform of a function $f(t)$, $t > 0$, is defined as

$$\mathcal{L}\{f(t)\} := \int_0^{\infty} f(t)e^{-st} dt.$$

Suppose $\mathcal{L}\{h(t)\} = \ln(1 + \frac{b^2}{s^2})$, b is some real number. Find $h(t)$.

7) Following the definition above find $\lim_{t \rightarrow 0^+} g(t)$ and $\lim_{t \rightarrow \infty} g(t)$ if

$$\mathcal{L}\{g(t)\} = \frac{16s^3 + 72s^2 + 216s - 128}{(s^2 + 2s + 5)^2}$$