

科目：工程數學乙 適用：電機所系統組

編號：431

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (20 %) The power method for an $n \times n$ matrix A can be performed by first selecting an arbitrary nonzero column vector \mathbf{x}_0 (having n components) and then executing the following iterations from $i = 1$.

Iteration i : Let $\mathbf{x}_i = A\mathbf{x}_{i-1}$ and compute $\frac{A\mathbf{x}_i \cdot \mathbf{x}_i}{\mathbf{x}_i \cdot \mathbf{x}_i}$.

Show that $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \dots$ converges to a dominant eigenvector and

$\frac{A\mathbf{x}_1 \cdot \mathbf{x}_1}{\mathbf{x}_1 \cdot \mathbf{x}_1}, \frac{A\mathbf{x}_2 \cdot \mathbf{x}_2}{\mathbf{x}_2 \cdot \mathbf{x}_2}, \dots$ converges to the dominant eigenvalue if A has n

linearly independent eigenvectors and \mathbf{x}_0 has a nonzero component in the direction of a dominant eigenvector.

2. (1) (10 %) Prove that similar matrices have the same eigenvalues.
(2) (10 %) Let A be an $n \times n$ matrix, derive the necessary and sufficient condition for A being diagonalizable.

3. (1) (10 %) Solve $\mathbf{X}' = \begin{pmatrix} 0 & 8 \\ 2 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{3t} \\ t \end{pmatrix}$ using undetermined coefficients.

- (2) (10 %) Solve $\mathbf{X}' = \begin{pmatrix} -5 & 3 \\ 2 & -10 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{-2t} \\ 1 \end{pmatrix}$ using variation of parameters.

4. (1) (10 %) Solve $\frac{dy}{dx} = \frac{y}{x + \sqrt{x^2 + y^2}}$, $y(0) = 1$.

- (2) (10 %) Solve $(\sin y + y \cos x) dx + (\sin x + x \cos y) dy = 0$.

5. Consider the L-R-C series circuit with $L = 1$ henry, $R = 40$ ohms, $C = 1/4000$ farads, and voltage source $E(t) = 24$ volts. Furthermore, assume there is zero initial current and zero initial charge.

- (1) (15 %) Determine the current as a function of time.

- (2) (5 %) Find the steady-state charge.

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